Appendix
Improving the Evaluation of Asset Pricing Models by Expanding the Set of Test Portfolios

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I The Cross-Section of Micro Portfolios

A Portfolio Formation Procedure

We describe the procedure for forming the set of micro portfolios in each size group (tiny-, small-, and big-cap). For each year t_a ($t_a = 1, ..., T_a$), we first sort the N_{t_a} stocks in a given size group according to their estimated average returns. To compute this variable denoted by $\hat{\mu}^s_{i,t_a}$ ($i = 1, ..., N_{t_a}$), we follow the approach of Fama and French (2006) in which the firm's average return is expressed as a linear combination of its book-to-market (BM), profitability (Y), and investment (A). For each month t prior to the formation date, we run a cross-sectional regression of the monthly stock excess returns on the most recent characteristics, $r_{i,t} = a_t + b_{BM,t} BM_{i,t} + b_{Y,t} Y_{i,t} + b_{A,t} A_{i,t} + e_{i,t}$, and then estimate the characteristic-based average return as

$$\hat{\mu}_{i,t_a} = a_{t_a} + b_{BM,t_a} BM_{i,t_a} + b_{Y,t_a} Y_{i,t_a} + b_{A,t_a} A_{i,t_a}, \tag{A1}$$

where BM_{i,t_a}, Y_{i,t_a}, and A_{i,t_a} are the characteristics observed in year t_a, and a_{t_a}, b_{BM,t_a}, b_{Y,t_a}, b_{A,t_a} are the time-series averages of the monthly coefficients. To faciliate the chaining of portfolio returns over consecutive years, we work with the standardized average return computed as

$$\hat{\mu}^s_{i,t_a} = \frac{\hat{\mu}_{i,t_a} - \frac{1}{N_{t_a}} \sum_i \hat{\mu}_{i,t_a}}{\left(\frac{1}{N_{t_a}} \sum_i \hat{\mu}^2_{i,t_a} - \left(\frac{1}{N_{t_a}} \sum_i \hat{\mu}_{i,t_a} \right)^2 \right)^{\frac{1}{2}}}. \tag{A2}$$

Second, we construct, for each stock i, a micro portfolio by equally weighting the stock itself and $n-1$ additional stocks with the nearest values to $\hat{\mu}^s_{i,t_a}$. The estimated average return of the newly-created portfolio, denoted by $\hat{\mu}^s_{i,t_a}(mp)$, is computed as $\frac{1}{n}\left(\hat{\mu}^s_{i,t_a} + \sum_m \hat{\mu}^s_{i_m,t_a}\right)$, where i_m ($m = 1, ..., n-1$) denotes the identity of the additional stocks included in the portfolio. This technique is called local averaging and borrows from Efron (2010, ch. 9).

Third, we chain the portfolio returns over time to obtain stable characteristic-based average returns. For each pair (i, j) of micro portfolios in years t_a and t_a+1, we compute the distance between them as $\left|\hat{\mu}^s_{i,t_a}(mp) - \hat{\mu}^s_{j,t_a+1}(mp)\right|$. Then, we match the portfolios with the lowest distance (each year-t_a portfolio can only be paired with one year-t_a+1 portfolio). To minimize changes in portfolio composition, we match the pair (i, i) first if $\left|\hat{\mu}^s_{i,t_a}(mp) - \hat{\mu}^s_{i,t_a+1}(mp)\right|$ is in the bottom 1% of all measured distances.

In Figure 1, we illustrate the portfolio formation procedure in a population of 50 stocks ($N_{t_a} = N = 50$) over a 2-year sample period ($T_a = 2$). Each dot denotes the ordered value $\hat{\mu}^s_{i,t_a}$ at the start of each year. We see that the portfolio composition changes each year to account for the time-variation in characteristics. For instance, the portfolio associated with the median average return $\hat{\mu}^s_{25,t_a}$ includes stocks S_{10}, $S_{42}, ...,$ S_3 in year 1, and stocks S_{18}, $S_6, ...,$ S_{46} in year 2. The formation procedure yields a total number of micro portfolios, M, equal to the number of stocks ($M = N$).

In practice, the formation procedure is more complicated because the number of stocks changes over time. Suppose that the number of stocks in year 2 is equal to 60 (instead of 50). Applying the matching procedure described above, we can pair 50 year-1 and 50 year-2 portfolios, which leaves 10 year-2 portfolios unmatched. In this example, the cross-section includes 60 micro portfolios ($M = 60$) with unequal time-series lengths: (i) 50 portfolios created in year 1 with complete return history (24 monthly returns), (ii) 10 unmatched portfolios created in year 2 with 12 monthly return observations. Conversely, suppose that we have 60 portfolios in year 1 (instead of 50). In this case, we can only pair 50 year-1 portfolios, which leaves 10 year-1 portfolios unmatched ($M = 60$). In general, the total number of micro portfolio is therefore equal to $M = \max_{t_a}(N_{t_a})$.

Please insert Figure 1 here

B Definition of the Characteristics

We use the definitions of Fama and French (2008, 2015) to compute the firm's book-to-market, profitability, and investment measures at the end of June of each year t_a. The book-to-market is equal to the ratio of the book value of equity to the market value of equity. The book value for year t_a is defined as total assets minus liabilities, plus balance sheet deferred taxes and investment tax credit (if available), minus preferred shares stock liquidating values (if available), or carrying value (if available). Each of these variables is computed using data in the fiscal year ending in the calendar year $t_a - 1$. The market value for year t_a equals the price times shares outstanding at the end of December of year $t_a - 1$. Profitability for year t_a is defined as revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by the book value of equity. Each of these variables is computed using data in the fiscal year ending in the calendar year $t_a - 1$. Finally, investment for year t_a is computed as the change in total assets between the fiscal years ending in calendar years $t_a - 2$ and $t_a - 1$. For each of these characteristics, we winsorize the data at the 1% and 99% levels to remove outliers.

II Estimation Procedure

A Extended Two-Pass Regression

We provide a general description of the econometric framework for estimating the pricing errors of the micro portfolios. Under the misspecified model m_κ, the excess return of each portfolio j ($j = 1, ..., M$) can be written as

$$r_{j,t} = a_j + b_{jm} r_{m,t} + b'_{j\kappa} f_{\kappa,t} + c'_j z_t + e_{j,t}, \tag{A3}$$

where $r_{m,t}$ is the market excess return, $f_{\kappa,t}$ is the K-vector of risk factors specific to model m_κ, z_t is a P-vector of mean-zero factors that capture the strong correlation structure between micro portfolios, and $e_{j,t}$ denotes the remaining residual term (weakly correlated). The intercept is equal to

$$a_j = \alpha_j - b'_{j\kappa} p^c_\kappa, \tag{A4}$$

where α_j is the portfolio pricing error, and p^c_κ is the K-vector of forward prices of the risk factors (i.e., their prices multiplied by the gross riskfree rate), defined such that the sum of squared alphas is minimized (e.g., Kan, Robotti, and Shanken (2013)).[1] Using Equation (A4), we can write the pricing error as

$$\alpha_j = \omega' \beta_j, \tag{A5}$$

where the $(K+1)$-vectors ω and β_j are defined as $\omega = [1, p^{c\prime}_\kappa]'$ and $\beta_j = (a_j, b'_{j\kappa})'$. To estimate α_j, we build on recent work by Gagliardini, Ossola, and Scaillet (2016; GOS hereafter) who extend the traditional two-pass regression to a large and unbalanced panel of test assets—two important features exhibited by micro portfolios.

In the first step, we run a time-series regression of $r_{j,t}$ on the $(K+P+2)$-vector $x_t = [1, r_{m,t}, f'_{\kappa,t}, z'_t]'$ for each portfolio j. The OLS estimator of the $(K+P+2)$-vector of coefficients $b_j = (a_j, b_{jm}, b'_{j\kappa}, c'_j)'$ is given by

$$\hat{b}_j = \left(\sum_{t=1}^{T} \mathit{1}_{j,t} x_t x'_t \right)^{-1} \sum_{t=1}^{T} \mathit{1}_{j,t} x_t r_{j,t}, \tag{A6}$$

where T is the total number of observations, and $\mathit{1}_{j,t}$ equals one if $r_{j,t}$ is non-missing

[1] The forward price of the market factor does not appear in equation (A4) because $r_{m,t}$ is an excess return which, by definition, has a forward price equal to zero ($p^c_m = 0$).

(and zero otherwise). The matrix inversion in Equation (A6) is numerically unstable if only few return observations are available. To address this issue, GOS introduce the following trimming device:

$$1_j^{\chi} = 1\left\{\tau_{j,T} \leq \chi_{1,T},\ CN(\hat{Q}_{x,j}) \leq \chi_{2,T}\right\}, \tag{A7}$$

where $\tau_{j,T} = \frac{T}{T_j}$, $T_j = \sum_{t=1}^{T} 1_{j,t}$, $CN(\hat{Q}_{x,j}) = \left(eig_{\max}(\hat{Q}_{x,j})/eig_{\min}\left(\hat{Q}_{x,j}\right)\right)^{\frac{1}{2}}$ denotes the condition number of the matrix $\hat{Q}_{x,j} = \frac{1}{T}\sum_{t=1}^{T} 1_{j,t} x_t x_t'$. Following GOS, we set $\chi_{1,T} = \frac{606}{60}$ (a minimum of 60 monthly observations) and $\chi_{2,T} = 15$.

In the second step, we estimate the K-vector of factor prices p_{κ}^c using a cross-sectional regression of the estimated intercept $\hat{a}_j$ on the K-vector of estimated betas $\hat{b}_{j\kappa}$ keeping the non-trimmed portfolios only:

$$\hat{p}_{\kappa(1)}^c = -\left(\sum_{j=1}^{M} 1_j^{\chi} \hat{b}_{j\kappa}' \hat{b}_{j\kappa}\right)^{-1} \sum_{j=1}^{M} 1_j^{\chi} \hat{b}_{j\kappa}' \hat{a}_j. \tag{A8}$$

We adjust $\hat{p}_{\kappa(1)}^c$ for the bias component $\Psi_{p_{\kappa}^c} = Q_{b_{\kappa}}^{-1}\left(\frac{1}{M}\sum_{j=1}^{M} \tau_{j,T} E_1' V \omega\right)$, where $Q_{b_{\kappa}} = E[b_{j\kappa}' b_{j\kappa}]$, $E_1 = [\mathbf{0}_{K\times 1}, I_K]'$, I_K is the $K \times K$ identity matrix, $V = E_2' Q_x^{-1} S_{jj} Q_x^{-1} E_2$, $S_{jj} = E[e_{j,t}^2 x_t x_t']$, and E_2 is a $(K+P+2) \times (K+1)$ matrix whose sth row e_s $(s = 1, 3, 4, ..., K+1)$ has zeros everywhere except for the sth element. The final estimate of p_{κ}^c is equal to

$$\hat{p}_{\kappa}^c = \hat{p}_{\kappa(1)}^c + \frac{1}{T}\hat{\Psi}_{p_{\kappa}^c}, \tag{A9}$$

where $\hat{\Psi}_{p_{\kappa}^c}$ is computed as $\hat{Q}_{b_{\kappa}}^{-1}\left(\frac{1}{M_{\chi}}\sum_{j=1}^{M} 1_j^{\chi} \tau_{j,T} E_1' \hat{V} \hat{\omega}_{(1)}\right)$, $M_{\chi} = \sum_{j=1}^{M} 1_j^{\chi}$, $\hat{Q}_{b_{\kappa}} = \frac{1}{M_{\chi}}\sum_{j=1}^{M} 1_j^{\chi} \hat{b}_{j\kappa}' \hat{b}_{j\kappa}$, $\hat{V} = E_2' \hat{Q}_{x,j}^{-1} \hat{S}_{jj} \hat{Q}_{x,j}^{-1} E_2$, and $\hat{\omega}_{(1)} = [1, \hat{p}_{\kappa(1)}^{c\prime}]'$. Following GOS, we estimate S_{jj} using the White estimator (1980): $\hat{S}_{jj} = \frac{1}{T}\sum_{t=l+1}^{T} 1_{j,t} \hat{e}_{j,t}^2 x_t x_t'$, where $\hat{e}_{j,t} = r_{j,t} - \hat{b}_j' x_t$. Plugging the estimated quantities in Equation (A5), we compute the pricing error as

$$\hat{\alpha}_j = \hat{\omega}' \hat{\beta}_j. \tag{A10}$$

B Estimation of the Portfolio t-Statistics

We now prove Proposition 1 which provides an analytical expression for the t-statistic associated with the pricing error and its asymptotic distribution.

Proof of Proposition 1. We consider the misspecified model m_k and suppose that the remaining residual terms $e_{j,t}$ $(j = 1, ..., M)$ are weakly correlated. When the number of portfolios and return observations grow large ($T, M \to \infty$), Proposition 7 of GOS shows that the estimated vector of forward prices converges towards p_κ^c at a rate equal to $\sqrt{M}$. In addition, standard results in regression analysis reveal that the vector of estimated coefficients $\hat{\beta}_j$ is asymptotically distributed as

$$\sqrt{T}(\hat{\beta}_j - \beta_j) \xrightarrow{d} N\left(0, V\right), \tag{A11}$$

where $V = E_2' Q_x^{-1} S_{jj} Q_x^{-1} E_2$. With M in the thousands and T in the hundreds, the asymptotic sampling variation in $\hat{\alpha}_j$ is therefore only driven by that of $\hat{\beta}_j$, i.e.,

$$\sqrt{T}(\hat{\alpha}_j - \alpha_j) \xrightarrow{d} N\left(0, \omega' V \omega\right). \tag{A12}$$

Using this result, we compute the portfolio t-statistic t_j as $\frac{\hat{\alpha}_j}{\hat{\sigma}_{\alpha_j}}$, where the estimated variance is given by

$$\hat{\sigma}_{\alpha_j}^2 = \frac{1}{T_j}\hat{\omega}'\hat{V}\hat{\omega} = \hat{\omega}'\hat{V}_{\beta_j}\hat{\omega}. \tag{A13}$$

The variance term $\hat{V}_{\beta_j}$ is equal to $\frac{1}{T_j}\hat{V}$, where $\hat{V}$ is a consistent estimator of the covariance matrix V. In addition, Equation (A12) implies that the t-statistic follows a normal distribution, $t_j \sim N\left(\frac{\alpha_j}{\sigma_{\alpha_j}}, 1\right)$, where $\alpha_j = \omega'\beta_j$ and $\sigma_{\alpha_j}^2 = \frac{1}{T}\omega' V \omega = \omega' V_{\beta_j} \omega$.

III Statistical Inference

A Proportion of Mispriced Portfolios

We compute the proportion of portfolios mispriced by model m_k as

$$\hat{\pi}_\kappa = 1 - \frac{\hat{G}_\kappa(A)}{F_0(A)} = 1 - \frac{\frac{1}{M}\sum_{j=1}^{M} \mathit{1}(t_j)}{\Phi_0(A)}, \tag{A14}$$

where $\hat{G}_\kappa(A)$ is the empirical t-statistic cdf, $\mathit{1}(t_j)$ is an indicator function equal to one if t_j falls in the interval A (and zero otherwise), and $F_0(A)$, is replaced with the standard normal cdf $\Phi_0(A)$ (as per Proposition 1). To conduct inference $\hat{\pi}_\kappa$, we rely on the following proposition.

Proof of Proposition 2. We consider the misspecified model m_k and suppose that the residual terms $e_{j,t}$ ($j = 1, ..., M$) are weakly correlated. We further assume that the t-statistics can be spatially ordered such that closely located t-statistics exhibit higher correlation.[2] When the number of portfolios grows large ($M \to \infty$), Lemma 2 of

[2] A natural variable for this spatial ordering is the characteristic-based average return because port-

Farcomeni (2006) shows that the empirical cdf $\hat{G}_\kappa(A)$ is normally distributed as

$$\sqrt{M}(\hat{G}_\kappa(A) - G_\kappa(A)) \xrightarrow{d} N\left(0, \sigma_\kappa^2\right), \tag{A15}$$

where $\sigma_\kappa^2 = var(\mathit{1}(t_1)) + 2\sum_{j=2}^{\infty} cov(\mathit{1}(t_1), \mathit{1}(t_j))$, and t_j $(j = 1, ..., \infty)$ are the ordered t-statistics.[3] Because the variance of the estimated proportion $\hat{\pi}_\kappa$ only depends on that of $\hat{G}(A)$ (see Equation (A14)), the asymptotic distribution of the vector of estimated proportions for two misspecified models m_{κ_1} and $m_{\kappa 2}$ is given by

$$\sqrt{M}\begin{bmatrix} \hat{\pi}_{\kappa 1} - \pi_{\kappa 1}^* \\ \hat{\pi}_{\kappa 2} - \pi_{\kappa 2}^* \end{bmatrix} \xrightarrow{d} N\begin{pmatrix} 0 & , & \frac{\sigma_{\kappa 1}^2}{\Phi_0^2(A)} & \frac{\sigma_{\kappa 1,\kappa 2}}{\Phi_0^2(A)} \\ 0 & & \frac{\sigma_{\kappa 2,\kappa 1}}{\Phi_0^2(A)} & \frac{\sigma_{\kappa 2}^2}{\Phi_0^2(A)} \end{pmatrix}. \tag{A16}$$

where $\pi_{\kappa 1}^* = E(\hat{\pi}_{\kappa 1})$ and $\pi_{\kappa 2}^* = E(\hat{\pi}_{\kappa 2})$. The variance terms are given by

$$\begin{aligned} \sigma_{\kappa 1}^2 &= var(\mathit{1}(t_1^{\kappa 1})) + 2\sum_{j=2}^{\infty} cov(\mathit{1}(t_1^{\kappa 1}), \mathit{1}(t_j^{\kappa 1})), \\ \sigma_{\kappa 2}^2 &= var(\mathit{1}(t_1^{\kappa 2})) + 2\sum_{j=2}^{\infty} cov(\mathit{1}(t_1^{\kappa 2}), \mathit{1}(t_j^{\kappa 2})), \\ \sigma_{\kappa 1,2} &= cov(\mathit{1}(t_1^{\kappa 1}), \mathit{1}(t_1^{\kappa 2})) + \sum_{j=2}^{\infty} cov(\mathit{1}(t_1^{\kappa 1}), \mathit{1}(t_j^{\kappa 2})) + cov(\mathit{1}(t_1^{\kappa 2}), \mathit{1}(t_j^{\kappa 1})), \end{aligned} \tag{A17}$$

where $t_j^{\kappa 1}$, $t_j^{\kappa 2}$ $(j = 1, ..., \infty)$ are the ordered t-statistics under models m_{κ_1} and $m_{\kappa 2}$.

Using Proposition 2, we can test the null hypothesis that model m_κ is correctly specified. Under the null hypothesis $H_0 : \pi_\kappa^* = 0$, the estimated mispricing proportion $\hat{\pi}_\kappa$ is asymptotically distributed as

$$\sqrt{M}\hat{\pi}_\kappa \xrightarrow{d} \frac{1}{2}\delta_0 + \frac{1}{2}N^+\left(0, \frac{\sigma_\kappa^2}{\Phi_0^2(A)}\right), \tag{A18}$$

where δ_0 is a point-mass at zero and N^+ is a positive-truncated normal distribution (see Proposition 3.2 of Genovese and Wasserman (2004)). To test this hypothesis at the size level ϕ, we determine whether $\hat{\pi}_\kappa$ is sufficiently far away from zero using the following threshold:

$$\hat{\pi}_\kappa > \chi_\phi \frac{1}{\sqrt{M}} \frac{\hat{\sigma}_\kappa}{\Phi_0(A)}, \tag{A19}$$

folios with similar average returns include similar stocks and are therefore more likely to have correlated returns.

[3] Equation (A15) extends the results of Genovese and Wasserman (2004) derived under the assumption that the t-statistics are independent.

where $\hat{\sigma}_\kappa$ is the consistent estimator of σ_κ, and χ_ϕ is the quantile of the standard normal distribution at (1-ϕ). To compute $\hat{\sigma}_\kappa$, we use the following estimator proposed by Newey-West (1987):

$$\hat{\sigma}_\kappa^2 = \left[\frac{1}{M}\sum_{j=1}^{M} \mathit{1}(t_j^\kappa)\right] - \hat{G}_\kappa^2(A) + 2\sum_{l=1}^{L}\left[\frac{1}{M-l}\sum_{j=1}^{M-l} \mathit{1}(t_j^\kappa)\mathit{1}(t_{l+j}^\kappa)\right] - \hat{G}_\kappa^2(A), \qquad \text{(A20)}$$

where L is set equal to 40 for the entire portfolio population (consistent with the results of our Monte-Carlo simulations presented below).

Proposition 2 also allows us test the null hypothesis of equal performance between two misspecified models (possibly non-nested). Under the null hypothesis $H_0 : \Delta\pi^* = \pi^*_{\kappa 1} - \pi^*_{\kappa 2} = 0$, the estimated difference $\Delta\hat{\pi}_\kappa = \hat{\pi}_{\kappa 1} - \hat{\pi}_{\kappa 2}$ is asymptotically distributed as

$$\sqrt{M}(\Delta\hat{\pi}_\kappa - \Delta\pi^*) \xrightarrow{d} N\left(0, \frac{\sigma^2_{\kappa 1} + \sigma^2_{\kappa 2} - 2\sigma_{\kappa 1,\kappa 2}}{\Phi_0^2(A)}\right). \qquad \text{(A21)}$$

To implement this testing procedure, we compute the covariance term $\sigma_{\kappa 1,\kappa 2}$ using the following consistent estimator:

$$\begin{aligned}\hat{\sigma}_{\kappa 1,2} &= \left[\frac{1}{M}\sum_{j=1}^{M} \mathit{1}(t_1^{\kappa 1})\mathit{1}(t_1^{\kappa 2})\right] - \hat{G}_{\kappa 1}(A)\hat{G}_{\kappa 2}(A) \\ &+ \sum_{l=1}^{L}\left[\frac{1}{M-l}\sum_{j=1}^{M-l} \mathit{1}(t_1^{\kappa 1})\mathit{1}(t_{l+j}^{\kappa 2})\right] - \hat{G}_{\kappa 1}(A)\hat{G}_{\kappa 2}(A) \\ &+ \sum_{l=1}^{L}\left[\frac{1}{M-l}\sum_{j=1}^{M-l} \mathit{1}(t_1^{\kappa 2})\mathit{1}(t_{l+j}^{\kappa 1})\right] - \hat{G}_{\kappa 1}(A)\hat{G}_{\kappa 2}(A), \qquad \text{(A22)}\end{aligned}$$

IV Extensions

A Sign of the Pricing Errors

We can extend the large-scale approach to conduct inference on the estimated proportions of portfolios with negative and positive pricing errors. To compute both proportions denoted by $\hat{\pi}_\kappa^-$ and $\hat{\pi}_\kappa^+$, we use the procedure of Barras, Scaillet, and Wermers (2010). First, we determine the proportions of portfolios with low or high estimated pricing errors by computing the t-statistic cdf $\hat{G}_\kappa$ over the intervals $A^- = [-\infty, -0.5]$ and $A^+ = [0.5, +\infty]$, respectively. Second, we deduct the proportion of "false discoveries", i.e., correctly-priced portfolios which, by chance, have t-statistics falling in the intervals A^-

and A^+. This two-step approach yields the following expressions for $\hat{\pi}_\kappa^-$ and $\hat{\pi}_\kappa^+$ and their variances:

$$\begin{aligned}\hat{\pi}_\kappa^- &= \hat{G}_\kappa(A^-) - (1-\hat{\pi}_\kappa)\Phi_0(A^-), \\ \hat{\pi}_\kappa^+ &= \hat{G}_\kappa(A^+) - (1-\hat{\pi}_\kappa)\Phi_0(A^+), \end{aligned} \tag{A23}$$

$$\begin{aligned}\sigma^2_{\pi_\kappa^-} &= \sigma^2_{G_\kappa(A^-)} + \Phi_0^2(A^-)\sigma^2_{\pi_\kappa} - 2\Phi_0(A^-)\sigma_{G_\kappa(A^-),\pi_\kappa}, \\ \sigma^2_{\pi_\kappa^+} &= \sigma^2_{G(A^+)} + \Phi_0^2(A^+)\sigma^2_{\pi_\kappa} - 2\Phi_0(A^+)\sigma_{G_\kappa(A^+),\pi_\kappa}, \end{aligned} \tag{A24}$$

where $\sigma^2_{G_\kappa(A^-)} = \frac{1}{M}\sigma^2_{\kappa^-}$, $\sigma^2_{G_\kappa(A^+)} = \frac{1}{M}\sigma^2_{\kappa^+}$, $\sigma^2_{\pi_\kappa} = \frac{1}{M}\frac{\sigma^2_\kappa}{\Phi_0^2(A)}$, $\sigma_{G_\kappa(A^-),\pi_\kappa} = \frac{1}{M\Phi_0(A)}\sigma_{\kappa^-,\kappa}$, and $\sigma_{G_\kappa(A^+),\pi_\kappa} = \frac{1}{M\Phi_0(A)}\sigma^2_{\kappa^+,\kappa}$. The different components are given by

$$\begin{aligned}\sigma^2_{\kappa^-} &= var(1^-(t_1^\kappa)) + 2\sum_{j=2}^{\infty} cov(1^-(t_1^\kappa), 1^-(t_j^\kappa)), \\ \sigma^2_{\kappa^+} &= var(1^+(t_1^\kappa)) + 2\sum_{j=2}^{\infty} cov(1^+(t_1^\kappa), 1^+(t_j^\kappa)), \\ \sigma_{\kappa^-,\kappa} &= cov(1^-(t_1^\kappa), 1(t_1^\kappa)) + \sum_{j=2}^{\infty} cov(1^-(t_1^\kappa), 1(t_j^\kappa)) + cov(1(t_1^\kappa), 1^-(t_j^\kappa)), \\ \sigma_{\kappa^+,\kappa} &= cov(1^+(t_1^\kappa), 1(t_1^\kappa)) + \sum_{j=2}^{\infty} cov(1^+(t_1^\kappa), 1(t_j^\kappa)) + cov(1(t_1^\kappa), 1^+(t_j^\kappa)), \end{aligned} \tag{A25}$$

where $1^-(t_j)$, $1^+(t_j)$ are indicator functions equal to one if t_j falls in the intervals A^- and A^+ (and zero otherwise). After replacing the above expressions with the consistent estimators proposed by Newey-West (1987) we can conduct inference on the two proportions $\hat{\pi}_\kappa^-$ and $\hat{\pi}_\kappa^+$.

B Testing for Useless Factors

We now explain how to test whether a given factor $f_{k,t}$ $(k = 1, ..., K)$ included in model m_κ is useless. For each portfolio j $(j = 1, ..., M)$, the beta on factor $f_{k,t}$ obtained from the first-pass regression in Equation (A6) is asymptotically distributed as

$$\sqrt{T}(\hat{b}_{jk} - b_{jk}) \xrightarrow{d} N\left(0, e_b' V e_b\right), \tag{A26}$$

where e_b is a $(K+1)$-vector with zeros everywhere except for the position associated with $\hat{b}_{jk}$. Using this result, we compute the associated t-statistic t_{jk} as $\frac{\hat{b}_{jk}}{\hat{\sigma}_{b_{jk}}}$, where the estimated variance of $\hat{b}_{jk}$ is given by

$$\hat{\sigma}_{b_{jk}} = \frac{1}{T_j} e_b' \hat{V} e_b. \tag{A27}$$

Then, we compute the proportion of portfolios with non-zero betas on factor $f_{k,t}$ using the same expression as in Equation (A14):

$$\hat{\pi}_{\kappa,b_k} = 1 - \frac{\hat{G}_{\kappa,b_k}(A)}{\Phi_0(A)} = 1 - \frac{\frac{1}{M}\sum_{j=1}^{M} \mathit{1}(t_{jk})}{\Phi_0(A)}, \tag{A28}$$

where $\hat{G}_{\kappa,b_k}$ is the empirical cdf of the beta t-statistics, and $\mathit{1}(t_{jk})$ is an indicator function equal to one if t_{jk} falls in the interval A (and zero otherwise).

If factor $f_{k,t}$ is useless, the *true* betas are all equal to zero (i.e., no beta dispersion) and the null hypothesis is defined as $H_0 : E(\hat{\pi}_{\kappa,b_k}) = \pi^*_{\kappa,b_k} = 0$. Based on Proposition 3.2 of Genovese and Wasserman (2004), we can write the distribution of the estimated proportion $\hat{\pi}_{\kappa,b_k}$ under H_0 as

$$\sqrt{M}\hat{\pi}_{\kappa,b_k} \xrightarrow{d} \frac{1}{2}\delta_0 + \frac{1}{2}N^+\left(0, \frac{\sigma^2_{\kappa,b}}{\Phi_0^2(A)}\right), \tag{A29}$$

where δ_0 is a point-mass at zero, N^+ is a positive-truncated normal distribution, and $\sigma^2_{\kappa,b}$ follows the same expression as in Equation (A17) except that we use the ordered beta t-statistics t_{jk}. To test this hypothesis at the size level ϕ, we determine whether $\hat{\pi}_{\kappa,b_k}$ is sufficiently far away from zero using the following threshold:

$$\pi^*_{\kappa,b_k} > \chi_\phi \frac{1}{\sqrt{M}} \frac{\hat{\sigma}_{\kappa,b_k}}{\Phi_0(A)}, \tag{A30}$$

where $\hat{\sigma}_{\kappa,b_k}$ is the consistent estimator of σ_{κ,b_k}, and χ_ϕ is the quantile of the standard normal distribution at (1-ϕ).

V Diagnostic Criterion for a Weak Correlation Structure

We use the approach developed by Gagliardini, Ossola, and Scaillet (2017) to test whether the remaining residual components $e_{j,t}$ $(j = 1, ..., M)$ are weakly cross-correlated. Similar to the extended two-pass regression presented above, this approach explicitly accounts for the large and unbalanced panel of micro portfolios. The criterion is given by

$$\xi = \mu_1 \left(\frac{1}{MT} \sum_{j=1}^{M} 1_j^{\chi} \bar{e}_j \bar{e}_j' \right) - g(M,T), \tag{A31}$$

where $\bar{e}_j$ is a T-vector containing the elements $1_{j,t} \hat{e}_{j,t}$ $(t = 1, ..., T)$, $\mu_1(X)$ is the largest eigenvalue of the matrix X, and $g(M,T)$ is a penalty function defined as

$$g(M,T) = \frac{\left(\sqrt{M} + \sqrt{T}\right)^2}{MT} \ln \left(\frac{MT}{\left(\sqrt{M} + \sqrt{T}\right)^2} \right). \tag{A32}$$

Under this approach, we validate the weak structure hypothesis if the estimated criterion $\hat{\xi}$ is negative. The rationale for this asymptotically-valid selection rule builds on Proposition 1 of Gagliardini, Ossola, and Scaillet (2017). When the number of portfolios and return observations grow large $(T, M \to \infty)$, this proposition shows that the probability that $\hat{\xi}$ is negative equals one under a weak factor structure. On the contrary, $\hat{\xi}$ is positive with probability one under a strong factor structure.

In our baseline specification, we compute the remaining residuals $\hat{e}_{j,t}$ $(j = 1, ..., M)$ using the P-vector z_t which contains the orthogonal components of the size, value, profitability, and investment factors of Fama and French (2015), and the equally-weighted returns of micro portfolios in the tiny- and big-cap groups (6 factors). For each proposed model, we find that $\hat{\xi}$ is negative and thus validate the weak structure hypothesis.

VI A Simple Illustrative Example

In Section II of the paper, we use a simple illustrative example to compare the performance of the two misspecified models m_a and m_b. We now explain how to compute the average value of the estimated mispricing proportion for each model m_κ $(\kappa = a, b)$:

$$E(\hat{\pi}_{\kappa,ta}) = \pi^*_{\kappa,ta} = 1 - \frac{G_{\kappa,ta}(A)}{F_0(A)}, \tag{A33}$$

where ta indicates the type of assets (ta =stocks, micro portfolios). To measure $G_{\kappa,ta}(A)$ and $F_0(A)$, we assume that the asymptotic theory provides a valid approximation of the distribution of the t-statistics of the pricing errors. This yields the following simple expressions:

$$G_{\kappa,stocks}(A) = \frac{1}{N}\sum_{i=1}^{N}\Phi_N\left(\frac{\alpha_i}{\sigma_{\alpha_i}};A\right),$$

$$G_{\kappa,portfolios}(A) = \frac{1}{M}\sum_{j=1}^{M}\Phi_N\left(\frac{\alpha_j}{\sigma_{\alpha_j}};A\right), \tag{A34}$$

where $\Phi_N(\frac{\alpha_i}{\sigma_{\alpha_i}};A)$ and $\Phi_N(\frac{\alpha_j}{\sigma_{\alpha_j}};A)$ are the cdfs of the normal distributions $N(\frac{\alpha_i}{\sigma_{\alpha_i}},1)$ and $N(\frac{\alpha_j}{\sigma_{\alpha j}},1)$ over the interval A, and N, M denote the total numbers of stocks and micro portfolios. The normality assumption also implies that the cdf for correctly-priced portfolios is given by

$$F_0(A) = \Phi_0(A), \tag{A35}$$

where $\Phi_0(A)$ is the standard normal cdf over the interval A.

For each individual stock i, we compute the pricing error under model m_κ as

$$\alpha_i = \mu_i - (b_{im}\lambda_m + b_{i\kappa}\lambda_\kappa^c), \tag{A36}$$

where λ_κ^c is the fitted premium obtained from the two-pass regression defined as $\frac{cov(\mu_i,b_{i\kappa})}{var(b_{i\kappa})}$. We further use the following approximation for the pricing error volatility,

$$\sigma_{\alpha_i} \approx \frac{1}{\sqrt{T_{stock}}}\sigma_{e_{stock}}, \tag{A37}$$

where T_{stock} is the number of return observations and $\sigma_{e_{stock}}$ is the residual volatility for individual stocks.

For each micro portfolio j, we have

$$\alpha_j = \mu_j - (b_{jm}\lambda_m + b_{j\kappa}\lambda_\kappa^c), \tag{A38}$$

and

$$\sigma_{\alpha_j} \approx \frac{1}{\sqrt{n}}\frac{1}{\sqrt{T_{port}}}\sigma_{e_{stock}}, \tag{A39}$$

where λ_κ^c is equal to $\frac{cov(\mu_i,b_{j\kappa})}{var(b_{j\kappa})}$, and n is the number of stocks in each portfolio ($n = 10$). We set both N and M equal to $4,500$. We further set $\sigma_{e_{stock}}$ equal to 0.12% per month, and T_{stock}, T_{port} equal to 190 and 390, respectively. These values correspond to the median levels across stocks and micro portfolios over our sample period.

VII Monte Carlo Analysis

A Setting

We conduct a Monte Carlo analysis to evaluate the finite-sample properties of the proportion estimators for two misspecified models m_a and m_b. We extend the simple illustrative example described above on several important dimensions to closely replicate the salient features of the data. First, we match the total number of micro portfolios across the three size groups (before imposing any filters on the data). Specifically, we construct a set of 2,349 tiny-cap portfolios, 938 small-cap portfolios, and 1,302 big-cap portfolios based on the empirical characteristics of the individual stocks in each size group.

Second, we account for the unbalanced nature of the panel of portfolio returns. To guarantee the same unbalanced structure as in the data, we apply the empirical $T \times M$ matrix of indicators $\mathit{1}_{j,t}$ ($j = 1, ..., M$ and $t = 1, ..., T$) to each simulated panel of portfolio returns, where T denotes the total sample size equal to 606 monthly observations, and M is equal to 4,589 micro portfolios.

Third, we jointly match the average proportion of mispriced portfolios across the proposed models examined in the empirical section by adding a size premium λ_s to the average excess return of each individual stock i ($i = 1, ..., M$):

$$\mu_i = b_{is}\lambda_s + b_{im}\lambda_m + b_{ia}\lambda_a + b_{ib}\lambda_b, \tag{A40}$$

where λ_m is the premium of the market return $r_{m,t}$, and λ_a, λ_b denote the premia of the two additional risk factors $f_{a,t}$ and $f_{b,t}$.

Model m_a includes the market and factor a, which implies that the vector of explanatory variables is defined as $x_{a,t} = [1, r_{m,t}, f_{a,t}, z_{b,t}]'$. The term $z_{b,t}$ is the estimated component of the omitted factor $f_{b,t}$ that is orthogonal to $x^0_{a,t} = [1, r_{m,t}, f_{a,t}]'$, i.e., $z_{b,t} = f_{b,t} - x^{0\prime}_{a,t}\hat{\beta}_{z_b}$, where $\hat{\beta}_{z_b}$ is the vector of estimated coefficients from a time-series regression of $f_{b,t}$ on $x^0_{a,t}$ over the entire sample period. Model m_b includes the market and factor b, which implies that $x_{b,t} = [1, r_{m,t}, f_{b,t}, z_{a,t}]'$, where $z_{a,t} = f_{a,t} - x^0_{b,t}\hat{\beta}_{z_a}$, $x^0_{b,t} = [1, r_{m,t}, f_{b,t}]'$, and $\hat{\beta}_{z_a}$ is the vector of estimated coefficients from a time-series regression of the omitted factor $f_{a,t}$ on $x^0_{b,t}$ over the entire sample period. We assume that $r_{m,t}$, $f_{a,t}$, $f_{b,t}$, and the residual term $e_{i,t}$ are all independent and normally distributed as $N(\lambda_m, \sigma^2_m)$, $N(\lambda_a, \sigma^2_a)$, $N(\lambda_b, \sigma^2_b)$, and $N(0, \sigma^2_e)$, respectively. We further assume that b_{is}, b_{im}, b_{ia}, b_{ib}, are randomly drawn from the normal distribution $N(E(b_s), var(b_s))$, $N(E(b_m), var(b_m))$, $N(E(b_a), var(b_a))$, $N(E(b_b), var(b_b))$.

To calibrate the model, we use monthly data on individual stocks (with a minimum of 100 observations) and the Fama-French three factors (market, size, value) over the entire sample period. The calibration of the distribution parameters for the betas and the residual term is done separately for each size group. To attribute each individual stock to a specific size group, we form, each year, the three size groups by taking as breakpoints the 20th and 50th percentiles of the market capitalization for the NYSE stocks (similar to Fama and French (2008)). We then classify each stock based on the frequencies at which it falls in the three groups. For each size group, we set $E(b_s)$ and $var(b_s)$ equal to the median and variance of the estimated size betas, and $E(b_m)$ and $var(b_m)$ equal to the median and variance of the estimated market betas. We further set $E(b_\kappa)$ and $var(b_\kappa)$ for each additional risk factor κ ($\kappa = a, b$) equal to the median and variance of the estimated value betas. Finally, σ_e is set equal to the cross-sectional average of the estimated residual volatility.

We set λ_s equal to 0.5% per month so as to approximate the median value for the proportions of mispriced portfolios (around 45%). We set λ_m and σ_m equal to the average return and volatility of the CRSP value-weighted index (0.5% and 4.4% per month). For the volatilities σ_a and σ_b of the additional risk factors a and b, we split the volatility of the value factor in two (2.8% per month). To determine the values for λ_a and λ_b, we choose two extreme scenarios to capture the minimum and maximum proportion differences observed in the data. Under the first scenario, λ_a and λ_b are set equal to 1.0% and 0.0% per month so as to produce a large proportion difference between the two models. Under the second scenario, λ_a and λ_b are both equal to 0.5% per month, which implies that both models yield the same moderate performance.

B Simulation Procedure

For each scenario, we compute the estimated proportions of mispriced portfolios over 1,000 iterations and five sets of values for the stock betas ($S = 5,000$). For each iteration s ($s = 1, ..., S$), we first construct a T-vector of monthly return observations for each stock i ($i = 1, ..., M$) :

$$r_{i,t}(s) = b_{is}\lambda_s + b_{im}r_{m,t}(s) + b_{ia}f_{a,t}(s) + b_{ib}f_{b,t}(s) + e_{i,t}(s), \tag{A41}$$

where $r_{m,t}(s)$, $f_{a,t}(s)$, $f_{b,t}(s)$, and $e_{i,t}(s)$ are drawn from their respective distributions. Second, we form the cross-section of micro portfolios using the average stock return μ_i as the sorting variable and apply the portfolio formation described in Section I of the appendix.[4] The resulting cross-section consists of M micro portfolios, each containing 10 stocks—stock i and nine additional stocks with the nearest average return to stock i. We keep track of the identity of the stocks included in each micro portfolio via a $M \times M$ matrix ID whose jth row id_j has zeros everywhere except in for the stocks included in the portfolio. Third, we construct the monthly return of each micro portfolio j from the M-vector of stock returns $r_t(s) = [r_{1,t}, ..., r_{M,t}]'$:

$$r_{j,t}(s) = 1_{j,t} \frac{1}{n} \left(id_j r_t(s) \right), \tag{A42}$$

where $1_{j,t}$ takes the value of one if the return is observed in the data (and zero otherwise). Fourth, we compute the vector of t-statistics for all portfolios using the extended two-pass regression described in Section II of the appendix and estimate the proportions of mispriced portfolios for the two models and its difference,

$$\begin{aligned}
\hat{\pi}_a(s) &= 1 - \frac{\hat{G}_a(A)(s)}{\Phi_0(A)}, \\
\hat{\pi}_b(s) &= 1 - \frac{\hat{G}_b(A)(s)}{\Phi_0(A)}, \\
\Delta\hat{\pi}(s) &= \hat{\pi}_a(s) - \hat{\pi}_b(s),
\end{aligned} \tag{A43}$$

as well as the estimated variances of these estimators using Equations (A20) and (A22),

$$\begin{aligned}
\hat{\sigma}^2_{\pi_a}(s) &= \frac{\hat{\sigma}^2_a(s)}{\Phi^2_0(A)}, \\
\hat{\sigma}^2_{\pi_b}(s) &= \frac{\hat{\sigma}^2_b(s)}{\Phi^2_0(A)}, \\
\hat{\sigma}^2_{\Delta\pi}(s) &= \frac{\hat{\sigma}^2_a(s) + \hat{\sigma}^2_b(s) - 2\hat{\sigma}_{a,b}(s)}{\Phi^2_0(A)}.
\end{aligned} \tag{A44}$$

Repeating these three steps S times, we can then compute the average values of the estimated proportions and their difference as

$$\begin{aligned}
E(\hat{\pi}_a) &= \pi^*_a = \frac{1}{S} \sum_{s=1}^{S} \hat{\pi}_a(s), \\
E(\hat{\pi}_b) &= \pi^*_b = \frac{1}{S} \sum_{s=1}^{S} \hat{\pi}_b(s),
\end{aligned}$$

[4] We assume that the book equity of each firm is proportional to its future expected cash flows. In this case, the average return can be directly inferred from the observable BM of each firm, i.e., μ_i is proportional to BM_i (see Berk (2000)).

$$E(\Delta\hat{\pi}) = \Delta\pi^* = \pi_a^* - \pi_b^*. \tag{A45}$$

We also compare the *true* variance of the estimators with the average estimated values:

$$\begin{aligned}
\sigma^2_{\pi_a} &= \frac{1}{S}\sum_{s=1}^{S}\hat{\pi}_a^2(s) - (\pi_a^*)^2 \quad \text{versus} \quad E(\hat{\sigma}^2_{\pi_a}) = \frac{1}{S}\sum_{s=1}^{S}\hat{\sigma}^2_{\pi_a}(s), \\
\sigma^2_{\pi_b} &= \frac{1}{S}\sum_{s=1}^{S}\hat{\pi}_b^2(s) - (\pi_b^*)^2 \quad \text{versus} \quad E(\hat{\sigma}^2_{\pi_b}) = \frac{1}{S}\sum_{s=1}^{S}\hat{\sigma}^2_{\pi_b}(s), \\
\sigma^2_{\Delta\pi} &= \frac{1}{S}\sum_{s=1}^{S}\Delta\hat{\pi}^2(s) - (\Delta\pi^*)^2 \quad \text{versus} \quad E(\hat{\sigma}^2_{\Delta\pi}) = \frac{1}{S}\sum_{s=1}^{S}\hat{\sigma}^2_{\Delta\pi}(s). \qquad \text{(A46)}
\end{aligned}$$

where we set $L = 40$, which represents 1% of the total sample.[5] To further measure the accuracy of the variance estimators, we compute the coverage ratio of the confidence intervals at ϕ equal to 90% and 95% as

$$\begin{aligned}
CR(\hat{\pi}_a) &= \frac{1}{S}\sum_{s=1}^{S} \mathit{1}\{abs(\hat{\pi}_a(s) - \pi_a^*) < \chi_\phi \hat{\sigma}_{\pi_a}(s)\}, \\
CR(\hat{\pi}_b) &= \frac{1}{S}\sum_{s=1}^{S} \mathit{1}\{abs(\hat{\pi}_b(s) - \pi_b^*) < \chi_\phi \hat{\sigma}_{\pi_b}(s)\}, \\
CR(\hat{\pi}_{\Delta\pi}) &= \frac{1}{S}\sum_{s=1}^{S} \mathit{1}\{abs(\Delta\hat{\pi}(s) - \Delta\pi^*) < \chi_\phi \hat{\sigma}^2_{\Delta\pi}(s)\}, \qquad \text{(A47)}
\end{aligned}$$

where $\mathit{1}\{.\}$ equals one if the condition inside the parenthesis is satisfied (and zero otherwise), and χ_ϕ equals the quantile of the standard normal distribution at (1-$\frac{\phi}{2}$).

C Main Results

In Panel A of Table AI, we examine the properties of the different estimators under the first scenario where the two models m_a and m_b achieve a large difference in performance (34.5% in the entire population). The *true* volatilities of the different estimators range between 3.6% and 9.0% and are typically higher for the two largest size groups which contain fewer portfolios. Turning to the properties of the variance estimators, we find that the average value for each model in the entire population is slightly below average (0.5% for model m_a and 0.3% for model m_b). In contrast, the volatility estimator for the difference yields an average value that closely matches the *true* volatility (5.1% versus 5.0%). This last property is maintained across all three size groups. Finally, the coverage

[5] For the small- and big-cap groups which only contain around 1,000 portfolios, we reduce L to 10.

ratios of the two confidence intervals at 90% and 95% are, in most cases, remarkably accurate. For instance, the coverage ratios for the proportion difference in the entire population are equal to 90.1% and 95.3%, respectively.

In Panel B, we repeat the analysis for the second scenario where the two models yield the same moderate performance. Similar to the previous scenario, the volatility estimators precisely capture the variability of the estimated mispricing proportions for the entire population (they are identical to the *true* values for both models). We also find that the coverage ratios stay close to their theoretical values (88.0% and 93.4% for the intervals at 90% and 95%, respectively). While the results are similar for the big-cap group, they are less accurate in the two smallest size groups (micro- and small-cap). In both groups, the volatility estimators underestimate the *true* volatilities by 12% on average (in relative terms), which implies that the coverage ratios of the confidence intervals are slightly lower than their theoretical values.

Please insert Table AI here

VIII Additional Results

A Change in the Number of Stocks

We now re-evaluate the different models after changing the number of stocks n included in each micro portfolio. In Panel A of Table AII, we report the estimated mispricing proportions for micro portfolios formed with 5 stocks ($n = 5$). The ranking of the different models is similar to the baseline case. In particular, the human capital CAPM performs well in each size group, the conditional CAPM produces low pricing errors in the two largest size groups, and the liquidity CAPM is successful among tiny-cap portfolios. However, the proportions of mispriced portfolios are lower than those reported in the baseline case, i.e., the average level in the entire population drops from 52.1% to 41.1%. With only 5 stocks in each portfolio, the benefits of diversification are not fully exploited and the detection of the mispriced portfolios becomes more difficult.

Next, Panel B repeats the analysis for micro portfolios formed with 15 stocks ($n = 15$). Compared to the baseline case, both the relative performance of the models and the mispricing proportions remain largely unchanged. In general, we also observe that the volatilities of the estimators are slightly higher. This is consistent with the fact that micro portfolios have a larger number of stocks in common and thus exhibit higher cross-correlation.

Please insert Table AII here

B Change in the Interval A

To compute the proportion of mispriced portfolios $\hat{\pi}_\kappa$, we have to specify the length of the interval A centered around zero (see Equation (A14)). We consider two alternative specifications where A is set equal to $[-0.25, 0.25]$ and $[-0.4, 0.4]$.[6] Table AIII shows that the mispricing proportions remain essentially unchanged, i.e., the averages are equal to 52.5% and 52.3% versus 52.1% in the baseline case. This lack of variation can be explained by the fact that both the numerator $\hat{G}_\kappa(A)$ and the denominator $\Phi_0(A)$ scale up and down with A, which leaves the ratio $\frac{\hat{G}_\kappa(A)}{\Phi_0(A)}$ nearly unchanged (a similar point is made by Barras, Scaillet, and Wermers (2010) and Storey (2002)).

Please insert Table AIII here

C Change in the Common Vector $\mathbf{z}_t$

We consider three alternative sets of common factors to capture the strong correlation structure between the portfolio residuals: (i) the Fama-French size, value, investment, and profitability factors, (ii) the Fama-French size and value factors, and (iii) no factors at all. Overall, the results in Table AIV are similar to those reported in the baseline case. With these new sets of common factors, the performance of the conditional CAPM and the human capital CAPM is even stronger—for instance, Panel B shows that the mispricing proportions in the big-cap group are respectively equal to 16.3% and 15.8% (versus 61.1% for the CAPM) when the size and value factors are used.

Contrary to our baseline specification, we find that the diagnostic $\hat{\xi}$ in Equation (A31) is positive in all three cases. Therefore, we reject the hypothesis that the remaining residuals $e_{j,t}$ $(j = 1, ..., M)$ are weakly correlated. This result has implications for statistical inference—the asymptotic distribution of the mispricing proportion and the expression for its volatility derived in Proposition 2 depend on the weak correlation assumption. Therefore, the estimated volatilities computed in Table AIV may be poor estimators of the *true* volatilities.[7]

Please insert Table AIV here

[6] In the baseline case, the interval $A = [-0.5, 0.5]$ includes 40% of the t-statistic observations for correctly-priced portfolios ($\Phi_0(A) = 0.40$). Under the two alternative scenarios, the values for $\Phi_0(A)$ are equal to 20% and 30%, respectively.

[7] This is likely to be the case in Panel C because no factors are used to capture the cross-correlation between the portfolio residuals.

D Bootstrap Analysis

In the baseline specification, we assume that the t-statistics of correctly-priced portfolios follow a standard normal distribution $N(0,1)$ (see Equation (A14)). We now relax this assumption by using the bootstrap approach of Efron (2010, ch. 2) in which the t-statistic of each portfolio is transformed into an alternative statistic called the z-value. This transformation guarantees that the z-value of a correctly-priced portfolio is distributed as a normal $N(0,1)$. Therefore, we can still use Equation (A14) to compute $\hat{\pi}_\kappa$ provided that we use z-values instead of t-statistics.

To compute the z-value of each portfolio j ($j = 1, ..., M$), we use the following procedure. First, we draw, for each bootstrap iteration s ($s = 1, ..., S$), random observations from the original sample of risk factors and residuals to re-construct the portfolio returns:

$$r_{j,t}(s) = a_{j,0} + \hat{b}_{jm} r_{m,t}(s) + \hat{b}_{j\kappa} f_{\kappa,t}(s) + \hat{c}_j z_t(s) + \hat{e}_{j,t}(s), \tag{A48}$$

where we impose that the portfolio is correctly priced ($\alpha_j = 0$) by setting

$$a_{j,0} = -\hat{b}_{j\kappa} \hat{p}^c_\kappa. \tag{A49}$$

Second, we re-estimate the portfolio t-statistic by regressing the bootstrapped returns on the bootstrapped factors, i.e.,

$$t_j(s) = \frac{\hat{\omega}' \hat{\beta}_j(s)}{\left(\hat{\omega}' \hat{V}_{\beta_j}(s) \hat{\omega}\right)^{\frac{1}{2}}}, \tag{A50}$$

where $\hat{\beta}_j(s) = (\hat{a}_j(s), \hat{b}_{j\kappa}(s))'$, $\hat{V}_{\beta_j}(s)$ denote the bootstrapped coefficient vector and its covariance matrix. Third, we repeat the first two steps S times and compute the bootstrapped cdf associated with the original t-statistic as

$$F_{0,j}(t_j) = \frac{1}{S} \sum_{s=1}^{S} 1\{t_j(s) \leqslant t_j\}. \tag{A51}$$

Finally, we obtain the z-value by inverting the quantile $F_{0,j}(t_j)$ using the standard normal cdf, i.e.,

$$z_j = \Phi_0^{-1}(F_{0,j}(t_j)). \tag{A52}$$

The bootstrap analysis based on 1,000 iterations ($S = 1,000$) is reported in Table AV. The estimated proportions of mispriced portfolios remain largely unchanged compared to the baseline case. This result implies that the sample size is sufficiently large for the normal distribution to be a good approximation for the *true* t-statistic distribution.

Please insert Table AV here

E Weighted Least Square Estimation

To account for the variation in the precision of the estimated coefficients $\hat{a}_j$ and $\hat{b}_j$ across micro portfolios in Equation (A8), we use a Weighted Least Square (WLS) approach to estimate the forward prices of the risk factors. Specifically, we follow GOS and multiply $\hat{a}_j$ and $\hat{b}_{j\kappa}$ by the weight $w_j = 1^{\chi}_j {v_j}^{-1}$, where v_j is the variance of the standardized errors $\sqrt{T}\hat{\alpha}_j = \sqrt{T}(\hat{a}_j + \hat{b}'_{j\kappa} p^c_\kappa)$ defined as

$$v_j = \tau_{j,T} \omega' V \omega. \tag{A53}$$

To compute the empirical counterpart of v_j, we use the following expression

$$\widehat{v}_j = \tau_{j,T} \hat{\omega}'_{(1)} \hat{V} \hat{\omega}_{(1)}. \tag{A54}$$

Using the estimated set of weights $\widehat{v}_j$, we estimate the vector of forward prices as

$$\hat{p}^c_\kappa = -\left(\sum_{j=1}^{M} \hat{w}_j \hat{b}'_{j\kappa} \hat{b}_{j\kappa} \right)^{-1} \sum_{j=1}^{M} \hat{w}_j \hat{b}'_{j\kappa} \hat{a}_j. \tag{A55}$$

Table AVI shows the results obtained with the WLS approach. While the mispricing proportions are slightly lower on average (45.9% versus 52.1% in the baseline case), the relative performance of the different models stays largely unchanged.

Please insert Table AVI here

F Exclusion of Financial Firms

In the baseline analysis, we include all common stocks traded in AMEX, NASDAQ, and NYSE. To examine whether our results are not driven by the specific characteristics of the financial industry, we follow Fama and French (2008) and remove all stocks with Standard Industrial Classification (SIC) codes between 6000 and 6999. The results in Table AVII reveal that the performance of the models remains largely unchanged.

Please insert Table AVII here

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Table AI
Monte Carlo Analysis

Panel A reports the properties of the proportion estimators under the first scenario where there is a large performance difference between the two misspecified models Ma and Mb. For the entire population and each size group (tiny-, small-, and big-cap), the first column shows the average values of the estimated proportions of mispriced portfolios for both models and their difference. The second and third columns compare the true volatilities of the estimated proportions and their difference with the estimated volatilities. The fourth and fifth columns show the coverage ratios of the confidence intervals at 90% and 95% for the estimated proportions and their difference. In Panel B, we repeat the analysis under the second scenario where the two models produce the same moderate performance. The total number of iterations is equal to 5,000.

Panel A: Large Performance Difference

		Volatility		Confidence Interval	
	Mean	True	Estimated	90%-coverage	95%-coverage
All Portfolios					
Model Ma	30.6	3.6	3.1	91.3	94.5
Model Mb	65.1	4.3	4.0	92.5	95.8
Difference	-34.5	5.0	5.1	90.1	95.3
Tiny-Cap Portfolios					
Model Ma	32.4	4.8	4.6	93.2	96.1
Model Mb	62.5	6.7	5.7	89.8	93.5
Difference	-30.1	8.0	7.7	87.9	93.7
Small-Cap Portfolios					
Model Ma	32.4	7.4	6.6	92.1	95.3
Model Mb	63.1	5.6	5.9	94.0	96.3
Difference	-30.6	9.0	9.1	90.3	94.8
Big-Cap Portfolios					
Model Ma	15.4	6.2	5.6	92.9	95.8
Model Mb	68.3	4.6	5.3	95.4	97.7
Difference	-52.9	7.3	7.6	91.0	95.5

Table AI

Monte Carlo Analysis (Continued)

Panel B: No Performance Difference

		Volatility		Confidence Interval	
	Mean	True	Estimated	90%-coverage	95%-coverage
All Portfolios					
Model Ma	40.0	2.9	2.9	94.1	96.9
Model Mb	40.0	2.9	2.9	94.1	96.7
Difference	0.0	4.0	3.8	88.0	93.4
Tiny-Cap Portfolios					
Model Ma	35.9	4.4	3.9	92.0	93.5
Model Mb	35.7	4.4	3.8	90.1	95.3
Difference	0.2	5.9	5.3	85.2	91.3
Small-Cap Portfolios					
Model Ma	33.3	6.5	5.8	90.0	93.3
Model Mb	33.0	6.3	5.8	92.7	95.3
Difference	0.3	9.5	8.2	84.4	90.3
Big-Cap Portfolios					
Model Ma	34.9	5.3	5.2	93.1	96.1
Model Mb	35.0	5.1	5.3	94.7	97.2
Difference	-0.1	7.4	7.2	88.7	93.6

Table AII

Change in the Number of Stocks

Panel A reports the estimated proportions of micro portfolios made up of five stocks (n=5) that are mispriced by the CAPM, the conditional CAPM, the human capital CAPM, the intertemporal CAPM, and the liquidity CAPM. This analysis is conducted for the entire portfolio population (All) and the three size groups (tiny-, small-, and big-cap). Figures in parentheses denote the estimated volatilities of the proportion estimates. In Panel B, we repeat the analysis using micro portfolios made up of 15 stocks (n=15).

Panel A: Five Stocks

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	51.1 (2.1)	61.1 (5.5)	48.4 (4.6)	31.7 (4.6)
Conditional CAPM	43.1 (1.9)	57.3 (5.5)	32.6 (3.8)	21.5 (4.4)
Human Capital CAPM	30.0 (2.8)	34.3 (4.2)	37.4 (5.1)	19.5 (4.7)
Intertemporal CAPM	46.6 (2.2)	51.1 (4.6)	52.1 (4.1)	34.6 (3.8)
Liquidity CAPM	34.3 (2.1)	40.6 (4.3)	43.4 (4.9)	22.7 (4.1)
Average	41.1	49.0	42.9	25.6

Panel B: Fifteen Stocks

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	74.2 (2.0)	80.5 (4.6)	75.5 (4.4)	59.4 (4.4)
Conditional CAPM	55.9 (2.2)	78.1 (4.9)	30.7 (4.8)	30.2 (5.2)
Human Capital CAPM	43.7 (3.4)	48.5 (4.7)	56.8 (5.8)	19.0 (4.9)
Intertemporal CAPM	66.7 (2.0)	69.3 (4.5)	76.1 (4.5)	57.7 (4.9)
Liquidity CAPM	36.4 (2.6)	39.7 (4.1)	52.4 (6.0)	35.1 (5.1)
Average	55.4	63.0	58.3	40.3

Table AIII

Change in the Interval A

Panel A reports the estimated proportions of micro portfolios that are mispriced by the standard CAPM, the conditional CAPM, the human capital CAPM, the intertemporal CAPM, and the liquidity CAPM using an interval A equal to [0.25,0.25]. This analysis is conducted for the entire portfolio population (All) and the three size groups (tiny-, small-, and big-cap). Figures in parentheses denote the estimated volatilities of the proportion estimates. In Panel B, we repeat the analysis using an interval A equal to [0.4,0.4].

Panel A: Interval A=[0.25,0.25]

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	64.6 (2.5)	76.1 (5.0)	60.0 (5.9)	43.9 (5.7)
Conditional CAPM	50.3 (2.6)	69.8 (5.7)	32.5 (4.7)	25.9 (5.4)
Human Capital CAPM	45.3 (3.1)	47.5 (5.4)	58.4 (5.8)	22.1 (6.1)
Intertemporal CAPM	64.0 (2.3)	62.1 (4.9)	70.8 (5.6)	50.0 (5.8)
Liquidity CAPM	43.6 (2.5)	40.9 (5.7)	49.8 (6.0)	30.7 (7.1)
Average	52.5	59.2	54.2	34.8

Panel B: Interval A =[0.4,0.4]

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	66.4 (2.2)	76.4 (5.2)	60.2 (4.7)	48.8 (4.7)
Conditional CAPM	51.2 (2.0)	70.1 (5.2)	29.3 (3.7)	22.4 (5.2)
Human Capital CAPM	44.7 (2.9)	47.0 (4.8)	52.3 (5.8)	20.3 (5.0)
Intertemporal CAPM	62.6 (2.1)	59.6 (5.0)	68.7 (4.6)	51.1 (4.5)
Liquidity CAPM	42.2 (2.4)	39.6 (5.0)	49.2 (5.2)	31.1 (6.0)
Average	52.3	58.6	51.9	34.2

Table AIV

Change in the Set of Common Factors

Panel A reports the estimated proportions of micro portfolios that are mispriced by the CAPM, the conditional CAPM, the human capital CAPM, the intertemporal CAPM, and the liquidity CAPM using as common factors the Fama-French size, value, investment, and profitability factors. This analysis is conducted for the entire portfolio population (All) and the three size groups (tiny-, small-, and big-cap). Figures in parentheses denote the estimated volatilities of the proportion estimates. In Panels B and C, we repeat the analysis using as common factors the Fama-French size and value factors and no factors at all.

Panel A: Size, Value, Investment, and Profitability Factors

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	64.3 (2.6)	73.4 (5.6)	60.5 (4.4)	47.9 (4.6)
Conditional CAPM	36.8 (2.3)	55.1 (4.5)	24.8 (4.1)	7.3 (4.5)
Human Capital CAPM	34.9 (3.3)	36.3 (5.2)	49.8 (5.3)	11.9 (5.2)
Intertemporal CAPM	59.3 (2.1)	67.0 (4.7)	63.8 (4.9)	57.5 (4.4)
Liquidity CAPM	33.1 (2.7)	37.2 (6.1)	50.7 (5.7)	27.0 (4.8)
Average	45.7	53.8	49.9	30.3

Panel B: Size and Value Factors

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	68.7 (2.4)	73.2 (5.4)	66.3 (4.8)	61.1 (4.4)
Conditional CAPM	35.6 (2.3)	49.1 (4.4)	29.3 (4.6)	16.3 (5.1)
Human Capital CAPM	39.8 (3.4)	34.0 (5.3)	57.7 (5.2)	15.8 (5.8)
Intertemporal CAPM	67.6 (2.2)	67.1 (4.6)	67.4 (4.8)	59.9 (4.2)
Liquidity CAPM	39.2 (2.8)	37.3 (5.6)	64.9 (5.0)	42.1 (5.6)
Average	50.3	52.2	57.1	39.1

Panel C: No Factors

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	61.4 (2.6)	68.6 (5.7)	55.7 (5.2)	51.1 (4.7)
Conditional CAPM	34.1 (2.6)	47.8 (4.7)	29.6 (5.7)	12.7 (4.6)
Human Capital CAPM	30.1 (3.9)	30.0 (6.0)	35.9 (5.4)	10.0 (4.9)
Intertemporal CAPM	62.0 (2.8)	65.1 (5.0)	56.6 (5.5)	55.0 (5.3)
Liquidity CAPM	27.3 (3.6)	23.5 (6.0)	51.5 (5.3)	41.1 (5.3)
Average	43.0	47.0	45.9	34.0

Table AIV
Bootstrap Analysis

This table reports the estimated proportions of micro portfolios that are mispriced by the CAPM, the conditional CAPM, the human capital CAPM, the intertemporal CAPM, and the liquidity CAPM using a bootstrap procedure to estimate the distribution of the t-statistics. This analysis is conducted for the entire portfolio population (All) and the three size groups (tiny-, small-, and big-cap). Figures in parentheses denote the estimated volatilities of the proportion estimates.

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	64.4 (3.1)	74.8 (5.4)	60.7 (4.2)	47.4 (4.2)
Conditional CAPM	47.5 (2.1)	71.2 (4.9)	27.3 (3.9)	20.2 (4.7)
Human Capital CAPM	40.7 (1.4)	45.5 (4.6)	50.4 (4.8)	19.2 (4.6)
Intertemporal CAPM	61.0 (1.4)	66.3 (4.7)	65.2 (4.9)	48.9 (4.0)
Liquidity CAPM	39.1 (1.3)	38.9 (5.0)	39.0 (5.4)	27.5 (5.1)
Average	50.5	59.3	48.5	32.6

Table AV

Weighted Least Square Estimation

This table reports the estimated proportions of micro portfolios that are mispriced by the CAPM, the conditional CAPM, the human capital CAPM, the intertemporal CAPM, and the liquidity CAPM using a Weighted Least Square (WLS) approach to estimate the price of the risk factor. This analysis is conducted for the entire portfolio population (All) and the three size groups (tiny-, small-, and big-cap). Figures in parentheses denote the estimated volatilities of the proportion estimates.

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	65.5 (2.1)	74.9 (5.4)	62.1 (4.0)	48.2 (4.2)
Conditional CAPM	47.5 (1.7)	64.5 (4.6)	30.1 (3.7)	24.6 (4.8)
Human Capital CAPM	37.3 (3.0)	39.0 (4.5)	46.5 (5.2)	25.6 (5.1)
Intertemporal CAPM	44.9 (2.6)	41.1 (4.7)	46.5 (5.4)	46.7 (4.4)
Liquidity CAPM	32.3 (2.6)	32.5 (4.6)	34.8 (6.1)	29.0 (5.0)
Average	45.5	44.0	34.8	45.5

Table AVI

Exclusion of Financial Firms

This table reports the estimated proportions of micro portfolios that are mispriced by the CAPM, the conditional CAPM, the human capital CAPM, the intertemporal CAPM, and the liquidity CAPM using non-financial firms only. This analysis is conducted for the entire portfolio population (All) and the three size groups (tiny-, small-, and big-cap). Figures in parentheses denote the estimated volatilities of the proportion estimates.

		Size Groups		
	All	Tiny-cap	Small-cap	Large-cap
CAPM	66.0 (2.2)	77.4 (5.3)	62.0 (3.8)	45.0 (4.6)
Conditional CAPM	55.3 (1.7)	74.8 (4.9)	37.1 (4.4)	20.9 (4.6)
Human Capital CAPM	42.3 (2.9)	46.2 (4.3)	56.3 (4.5)	12.7 (4.9)
Intertemporal CAPM	60.2 (2.2)	62.9 (4.7)	65.2 (5.1)	45.0 (4.4)
Liquidity CAPM	36.1 (2.5)	43.8 (4.2)	57.7 (3.9)	25.1 (4.3)
Average	52.0	61.0	55.7	29.8

Figure 1

Forming the Cross-Section of Micro Portfolios

This figure illustrates the procedure for forming the set of micro portfolios sorted based on average returns using an hypothetical population of 50 individual stocks and a two-year sample period. Each dot represents the estimated average return taken by each stock (S1, S2,...). This value is standardized each year by removing the cross-sectional average and dividing by its standard deviation. For each stock, the procedure consists of forming an equally-weighted portfolio that includes the stock itself and 9 additional stocks with the nearest estimated average returns. Then, the portfolio returns are chained across years 1 and 2 to maintain a stable average return over time. This procedure yields a cross-section of 50 micro portfolios ranging from P(Return 1) to P(Return 50).